Chapter 15
Signal, Contrast and Noise (part 1)

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- **Previous classes:**
  - Signal detection (Chaps. 3-7)
  - T1, T2 and T2*, GE and SE (chap.8)
  - Signal sampling, DFT (Chaps. 9-12)
- **Today's content**
  - Signal and noise in MRI
  - SNR dependence on imaging parameters

Intro

- Noise is practically inevitable
  - Random / systematic
  - Physiological

- Signal-to-noise ratio (SNR) determines the effectiveness of the imaging

- SNR affects CNR, which determines the usefulness of the image

- Good CNR requires good SNR, but good SNR not necessary means good CNR

Signal and noise

- Revisit the measured spin density

1D: $\hat{\rho}_m(q\Delta x) = \frac{1}{\mathrm{N}}\sum_{p=-\frac{N}{2}}^{\frac{N}{2}-1} s(p\Delta k)e^{i2\pi pq/N}$ (Eq. 12.36)

$= \Delta x \sum_{q=-\frac{N}{2}}^{\frac{N}{2}-1} \hat{\rho}(q\Delta x)\delta(x - q\Delta x)$ (Eq. 12.25)

3D: $\hat{\rho}_m(p\Delta x, q\Delta y, r\Delta z) = \frac{1}{N_x N_y N_z}\sum_{p',q',r'} s(p'\Delta k_x, q'\Delta k_y, r'\Delta k_z)e^{i2\pi\left(\frac{pp'}{N_x}+\frac{qq'}{N_y}+\frac{rr'}{N_z}\right)}$

Voxel Signal

$= \boxed{\Delta x \Delta y \Delta z}\sum_{p,q,r} \hat{\rho}(p\Delta x, q\Delta y, r\Delta z)\delta(x - p\Delta x)\delta(y - q\Delta y)\delta(z - r\Delta z)$

Voxel Volume

Signal and noise

- Consider equilibrium magnetization and receive coil field

$$s \equiv \hat{\rho}_m \propto \omega_0 M_0 \mathcal{B}_\perp V_{voxel}$$
$$\propto \gamma B_0 \rho_0 \frac{\gamma^2 h^2}{4kT} B_0 \mathcal{B}_\perp V_{voxel}$$
$$\propto \frac{B_0^2 V_{voxel}}{T} \rho_0 \mathcal{B}_\perp$$

- Many factors being ignored or constant here
  - T1, T2, T2*
  - RF field: flip angle, B1 field homogeneity
  - Sequence type, TE/TR/TI/…

Signal and noise

- Noise in MRI
  - Thermal noise (body/object, coil, electronics)
  - Systematic noise (scanner, AC power, etc)
  - Physiological noise (heartbeat, respiration, etc)

- Thermal noise estimation

$$\sigma_{thermal}^2 \propto 4kT \cdot R \cdot BW$$

$$R_{eff} = R_{body} + R_{coil} + R_{electronics}$$

BW: bandwidth of reception

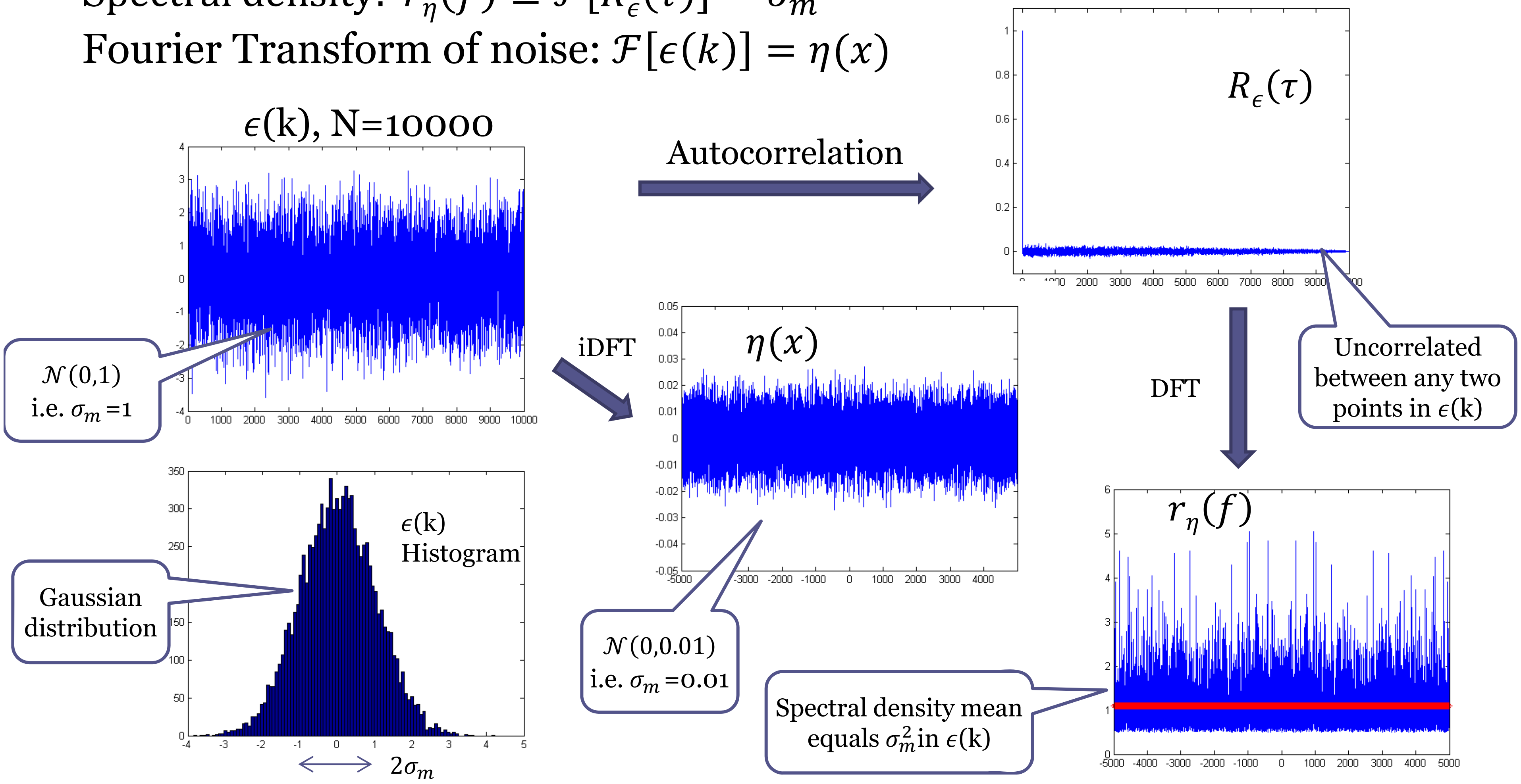
$$s_m(k) = s(k) + \epsilon(k)$$

- For thermal (or white/Gaussian) noise

Autocorrelation: $R_\epsilon(\tau) \equiv \overline{\epsilon(k_p)\epsilon^*(k_q)}|_{\tau\equiv(k_p-k_q)} = \sigma_m^2\delta(\tau)$

Spectral density: $r_\eta(f) \equiv \mathcal{F}[R_\epsilon(\tau)] = \sigma_m^2$

Fourier Transform of noise: $\mathcal{F}[\epsilon(k)] = \eta(x)$



Noise vs. Imaging parameters

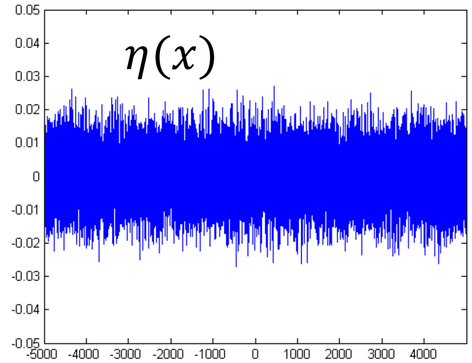
- Noise in MR image

$$\eta(p\Delta x) = iDFT[\epsilon(k)] = \frac{1}{N}\sum_{p'}\epsilon(p'\Delta k)e^{i2\pi p'\Delta kp\Delta x}$$

- Properties of noise in image

$$mean\big(\eta(p\Delta x)\big) = iDFT\big[\overline{\epsilon(k)}\big] = 0$$

$$var\big(\eta(p\Delta x)\big) \equiv DFT[R_\epsilon(\tau)] = \frac{\sigma_m^2}{N}$$



- Implications
  - White noise in k-space results in white noise in image
  - Noise variance (std) in image is N ($\sqrt{N}$) times smaller than in k-space

Improving SNR

- Averaging over multiple repeated acquisitions

$$s_{m,ave}(k) = \frac{1}{N_{acq}} \sum_i s_{m,i}(k) = s_m(k)$$

$$\sigma_{m,ave} = \frac{1}{N_{acq}} \sqrt{\sum_i \sigma_{m,i}^2} = \frac{\sigma_m}{\sqrt{N_{acq}}}$$

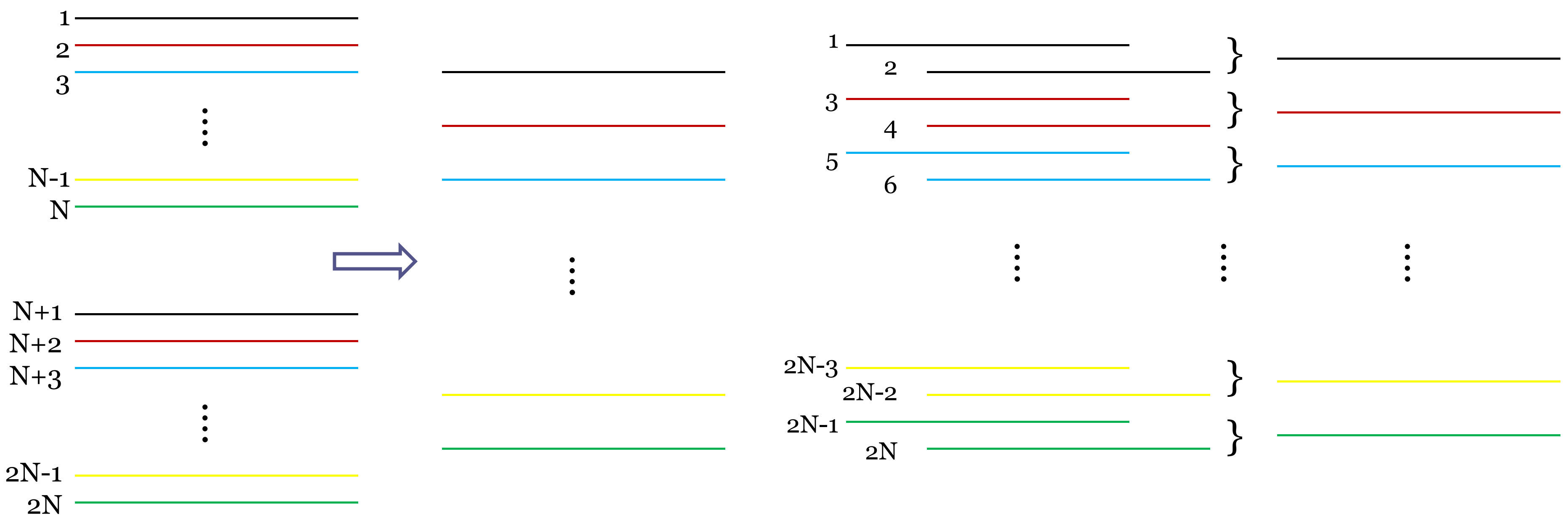
Thus

$$SNR_{m,ave} = \sqrt{N_{acq}} SNR_m$$

- Properties
  - True for both k-space and images
  - Valid only when noise are uncorrelated between acquisitions, systematic noise such as artifacts will not be reduced

Improving SNR

- Averaging can be done over images or k-space
- Two modes for averaging over k_y

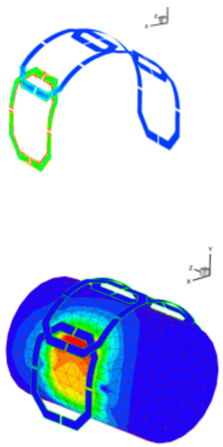


Long term ave

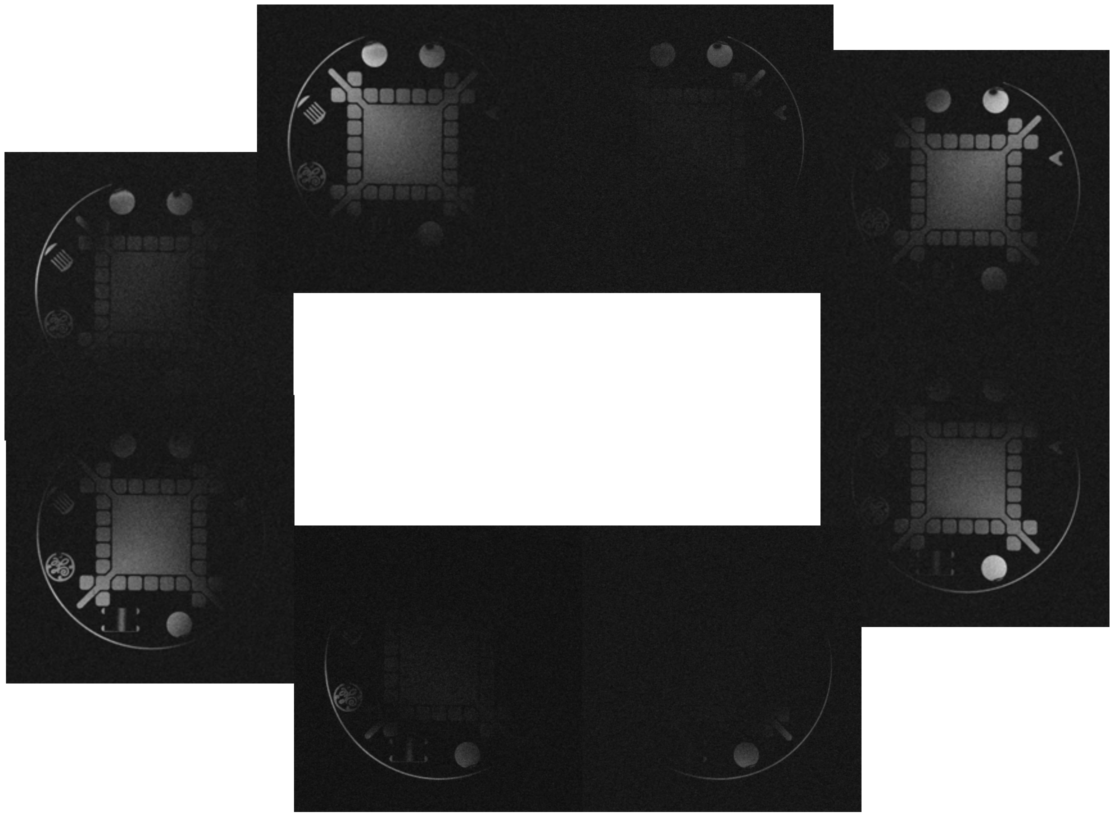
Short term ave

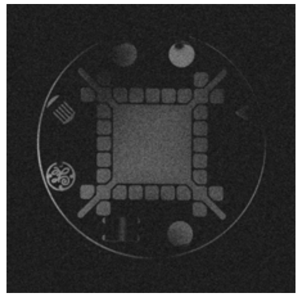
Improving SNR

- Use multiple smaller surface coils (phased array coils)
  - Smaller coil -> less coupling -> less noise
  - Rapidly weakening receive fields $\mathcal{B}$ with distance
  - Sum-of-square offers optimal results over linear combination

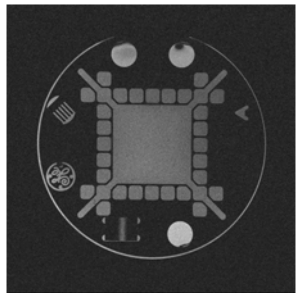


http://www.insightmri.com/





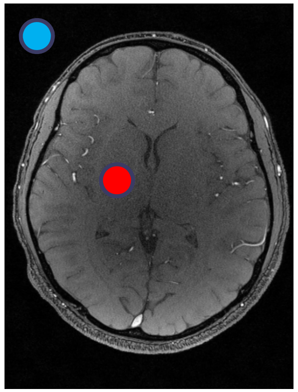
Linear

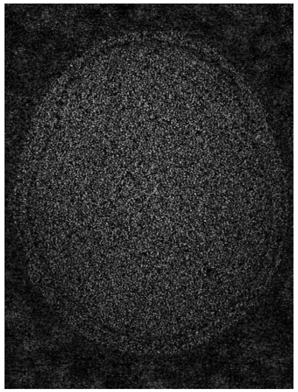


SOS

Measuring SNR

- Mean and std of ROI in homogeneous regions
- Use std in background ROI to estimate σ_o ($\sigma_{bg}\quad 0.655\,\sigma_o$, Rayleigh distribution)
- $\sigma_{oj} > \sigma_{bg}$
  Gaussian vs. Rayleigh; physiological noise
- Scan twice, add and subtract the two images to get mean and std ($\sigma_{sub}\quad \sqrt{2}\sigma_o$)
- Collect multiple volumes of images for voxel wise SNR (tSNR)

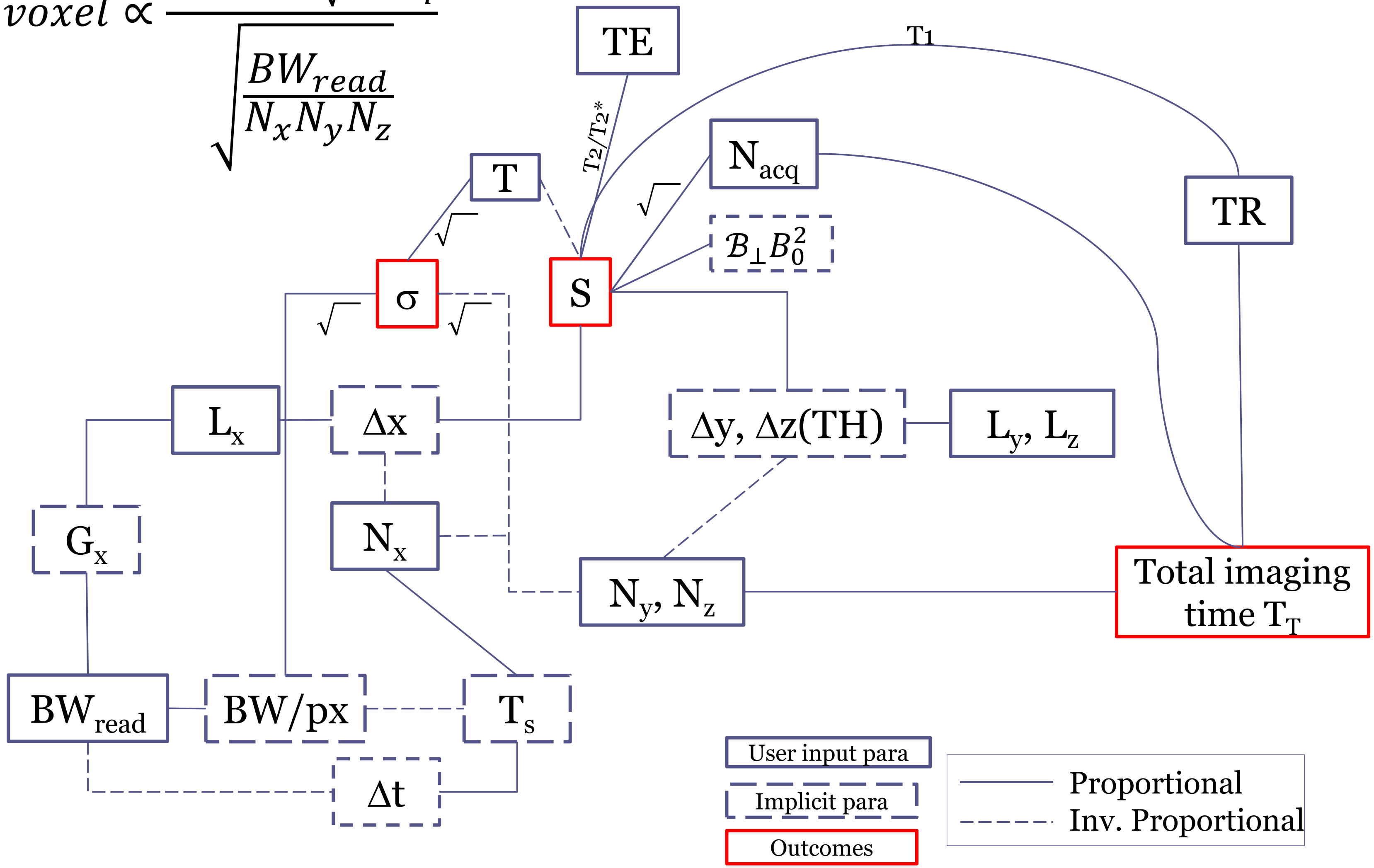




subtracted

SNR dependence on imaging parameters

$$SNR/voxel \propto \frac{\Delta x \Delta y \Delta z \sqrt{N_{acq}}}{\sqrt{\frac{BW_{read}}{N_x N_y N_z}}}$$



Imaging efficiency

- Total imaging time

$$T_T = N_{acq} N_y N_z TR$$

- Imaging efficiency (with otherwise fixed para)

$$\Upsilon \equiv \frac{SNR/voxel}{\sqrt{T_T}} \propto \Delta x \Delta y \Delta z \sqrt{T_s}$$

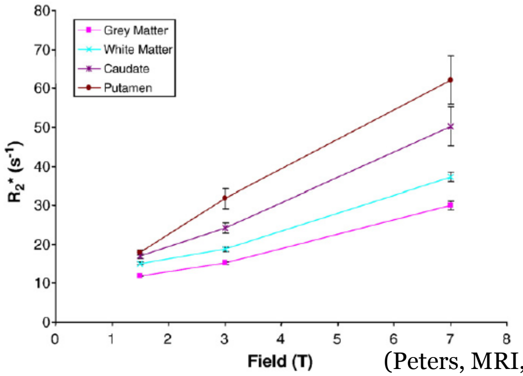
- Implications
  - Large voxels yield better SNR efficiency
  - So does longer readout (or lower RO bandwidth). But sometimes T2* decay may negates the effects

SNR vs. B_0

- B_0 affects MRI signal and noise in a number of ways, e.g. M_0, T1, T2, field inhomogeneity, and thus T2*
  - $S \propto \frac{B_0^2 V_{voxel}}{T} \propto \omega_0^2$
  - $R_{eff}(\omega_0) \approx \begin{cases} R_{coil}(\omega_0) + R_{electronices}(\omega_0) \\ R_{sample}(\omega_0) \end{cases} \propto \begin{cases} \sqrt{\omega_0}, & \omega_0 \ll \omega_{0,mid} \\ \omega_0^2, & \omega_0 \gg \omega_{0,mid} \end{cases}$

$$SNR(\omega_0) \propto \begin{cases} \omega_0^{7/4} \\ \omega_0 \end{cases} \qquad \Leftarrow \qquad \sigma_{thermal} \propto \begin{cases} \omega_0^{1/4}, & \omega_0 \ll \omega_{0,mid} \\ \omega \end{cases}$$

- Higher B_0 -> longer T1, shorter T2/T2*



(Peters, MRI,2007)

Homework

- Prob 15.1, 15.3, 15.4, 15.8, 15.10, 15.12, 15.13

Next Session

Chapter 15.3-15.6

Chapter 15
Signal, Contrast and Noise (part 2)

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- **Previous sessions:**
  - T1, T2 and T2* , IR (Chap.8)
  - Signal sampling, DFT (Chaps. 9-12)
  - Signal and noise (Chaps. 15.1-15.2)
- **Today's content**
  - Contrast and noise in MRI
  - Contrast mechanisms and enhancement
  - Partial volume effects, CNR and resolution

Contrast, CNR and visibility

- High SNR is only half the story

- The other half is to be able to distinguish different object/structure/properties, i.e. have good enough contrast with the presence of noise

Contrast, CNR and visibility

- Contrast: simply the signal difference

$$C_{AB} \equiv S_A - S_B$$

- CNR: contrast-to-noise ratio

$$CNR_{AB} \equiv SNR_A - SNR_B$$

when $\sigma_A = \sigma_B = \sigma_0$,

$$CNR_{AB} = \frac{C_{AB}}{\sigma_0}$$

Contrast, CNR and visibility

- Visibility

$$\mathcal{V}_{AB} \equiv \frac{C_{AB}}{\sigma_{eff}} = \frac{C_{AB}}{\sigma_0}\underbrace{\sqrt{N_{acq}}}_{\text{Temporal}} = \frac{C_{AB}}{\sigma_0}\underbrace{\sqrt{n_{voxel}}}_{\text{Spatial}}$$

- The Rose Criterion

$$CNR_{AB} > 3\sim5$$

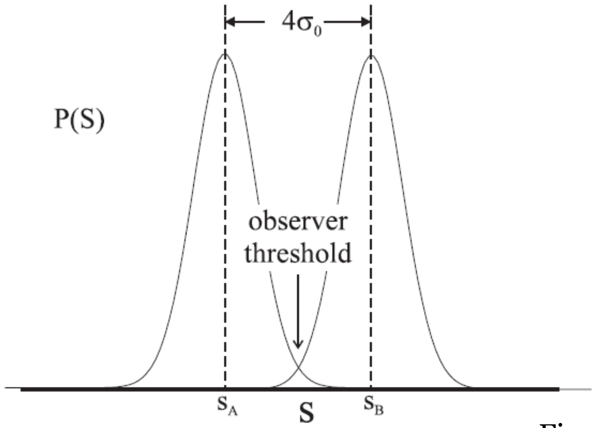


Fig.15.4

Contrast, CNR and visibility

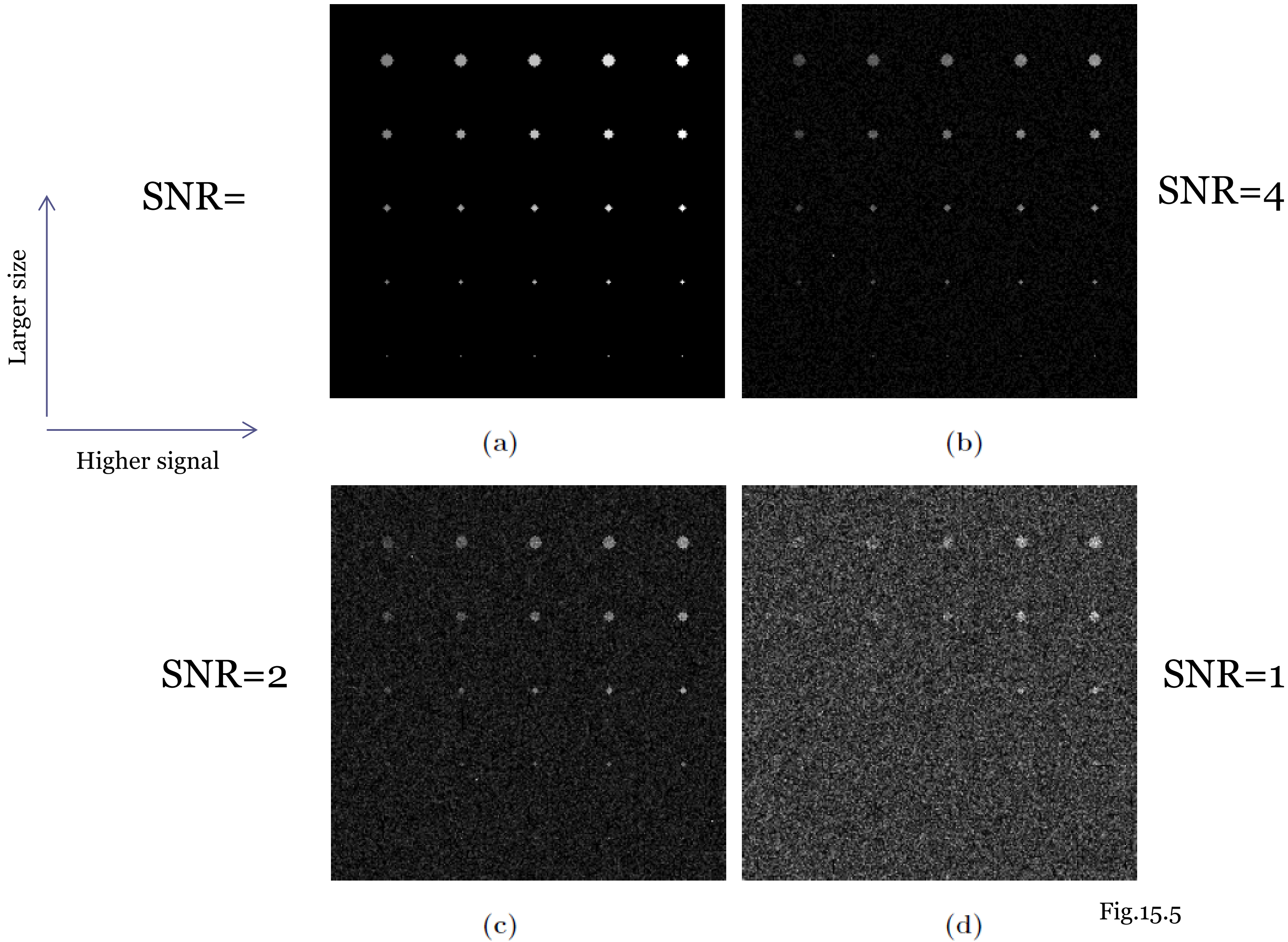


Fig.15.5

Contrast mechanisms in MRI

- MRI is a multiple contrast imaging modality

Intrinsic:
- spin density
- T1/T2/T2*/T1ρ
- Diffusion
- Chemical shift
- Susceptibility
- Temperature
- …

Physiological:
- Blood flow
- Perfusion
- Blood oxygenation
- Tissue elasticity
- …

Sequence specific:
- Steady state (chap.18)
- Selective excitation /suppression
- …

- Or multiple methods for the same contrast
  - Example:
    T1W : short TE/TR GE, or IR
    Signal nulling: IR, selective excitation/saturation, dephasing

Most important contrast types

- ρ_0, T1 and T2 (T2*)

$$S_A(TR,TE) = \rho_{0,A}(1 - e^{-TR/T_{1,A}})e^{-TE/T^*_{2,A}}$$

$$S_B(TR,TE) = \rho_{0,B}(1 - e^{-TR/T_{1,B}})e^{-TE/T^*_{2,B}}$$

$$\Downarrow$$

$$C_{AB}(TR,TE) = S_A(TR,TE) - S_B(TR,TE)$$

Most important contrast types

$$S(TR,TE) = \rho_0(1 - e^{-TR/T_1})e^{-TE/T_2^*}$$

- Spin density weighting

  TR T1 => e^{-TR/T_1} ->0

  TE T2* => e^{-TE/T_2^*} ->1

$$C_{AB}(TR,TE) \approx \rho_{0,A} - \rho_{0,B} +$$

Note: $\rho_{0,A}$ and $\rho_{0,B}$ are effective spin density

Most important contrast types

$$S(TR, TE) = \rho_0(1 - e^{-TR/T_1})e^{-TE/T_2^*}$$

- T1 weighting

use TE T2* => e^{-TE/T_2^*} ->1

$$C_{AB}(TR) \approx \rho_{0,A} - \rho_{0,B} - (\rho_{0,A}e^{-\frac{TR}{T_{1,A}}} - \rho_{0,B}e^{-\frac{TR}{T_{1,B}}})$$

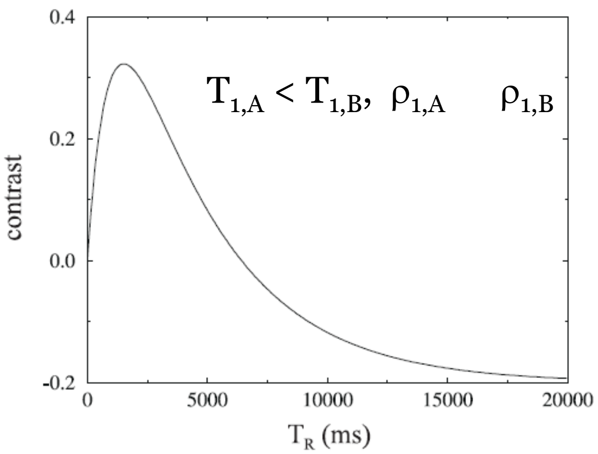


Fig. 15.7b

$$TR_{opt} = \frac{\ln\left(\frac{\rho_{0,B}}{T_{1,B}}\right) - \ln\left(\frac{\rho_{0,A}}{T_{1,A}}\right)}{\frac{1}{T_{1,B}} - \frac{1}{T_{1,A}}}$$

$$= T_{1,A}\big|_{T_{1,A} \cong T_{1,B}}$$

T1 values at 3T
GM : ~1200ms
WM: ~800ms
CSF: ~2-4s

Most important contrast types

$$S(TR, TE) = \rho_0(1 - e^{-TR/T_1})e^{-TE/T_2^*}$$

- T2* weighting

use TR T1 => e^{-TR/T_1} ->0

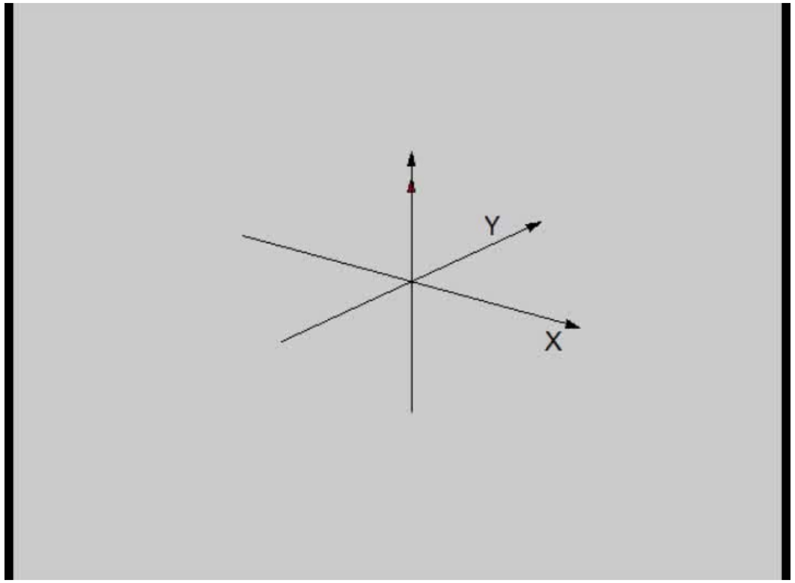
$$C_{AB}(TE\ ) = \rho_{0,A} e^{-\frac{TE}{T_{2,A}^*}} - \rho_{0,B} e^{-\frac{TR}{T_{2,B}^*}}$$

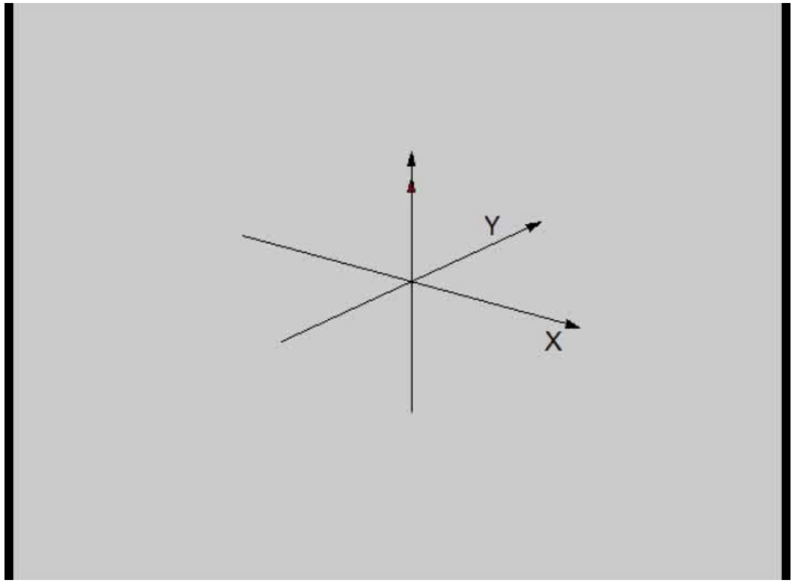
$$TE_{opt} = \frac{\ln\left(\frac{\rho_{0,B}}{T_{2,B}^*}\right) - \ln\left(\frac{\rho_{0,A}}{T_{2,A}^*}\right)}{\frac{1}{T_{2,B}^*} - \frac{1}{T_{2,A}^*}}$$
$$= T_{2,A}^*|_{T_{2,A}^* \cong T_{2,B}^*}$$

T1 weighting with inversion recovery (Chap 8.4)

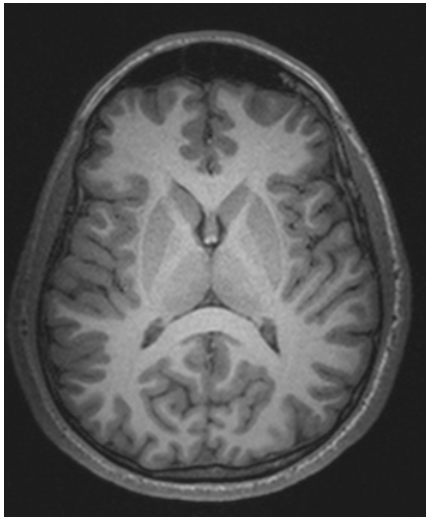
$$M_\perp(TI, TE) = M_0\left(1 - 2e^{-\frac{TI}{T1}}\right)e^{-\frac{TE}{T_2{}^*}}$$

- IR may double the T1 contrast relative to GE
- Optimal TI (?)
- Potential of nulling certain tissues

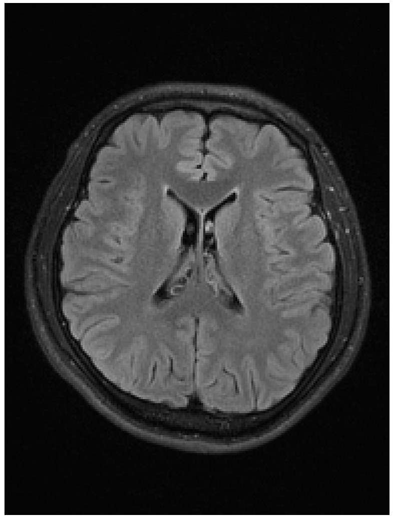




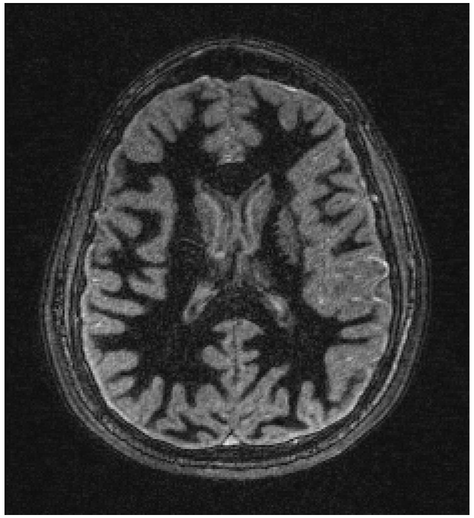
IR examples



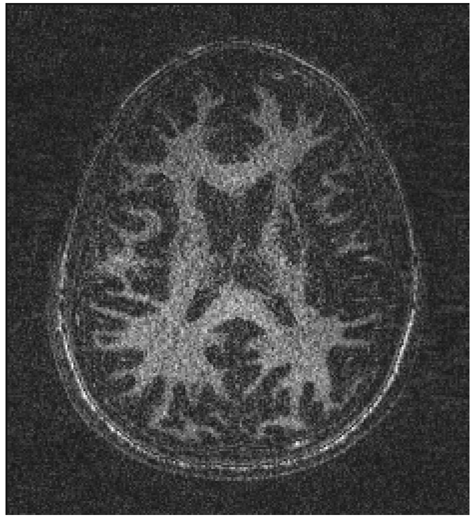
T1W



CSF nulling, T2W

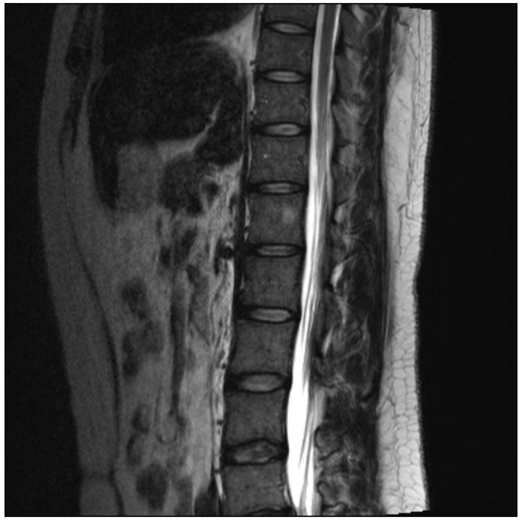


WM+CSF nulling



GM+CSF nulling

T2W

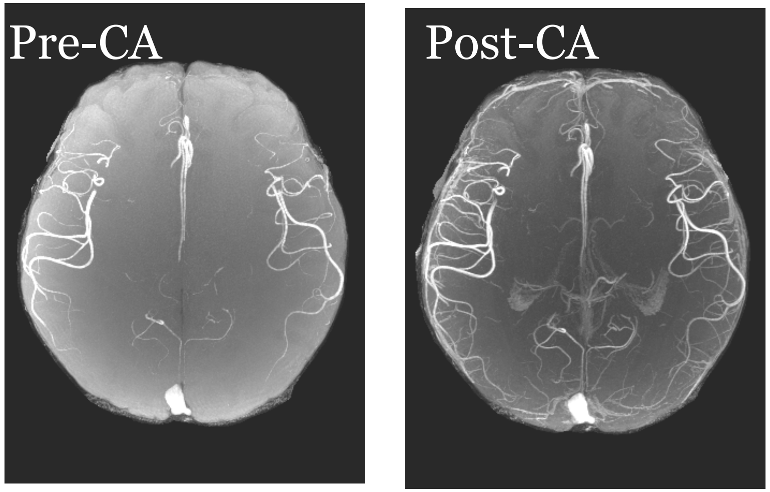


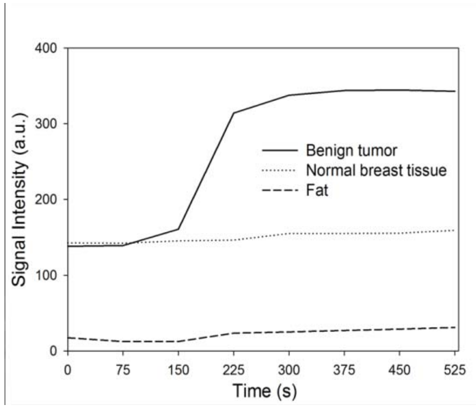
Fat nulling,T2W

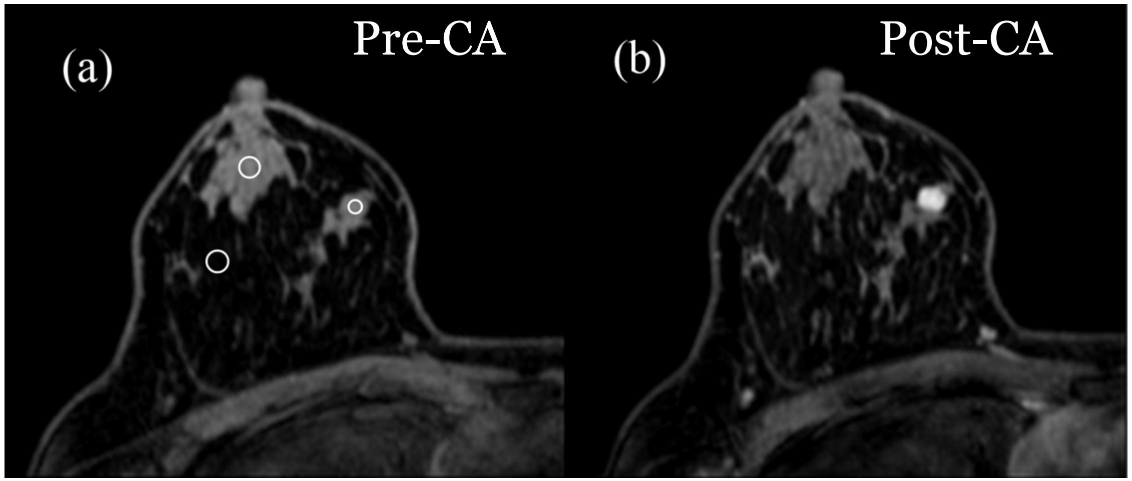
Contrast enhancement with T1-shortening agents (Gd)

- With T1 shortened , target tissue will have increased signal in short TR GE images
- T1 shortening effect proportional to agent concentration
- Target tissue example:
  - Blood (MR angiography, perfusion weighted imaging)
  - Tumor
  - Vessel wall lesion

Contrast enhanced images







CNR, partial volume and resolution

$$\mathcal{V}_{AB} \equiv \frac{C_{AB}}{\sigma_{eff}} = \frac{C_{AB}}{\sigma_0}\sqrt{N_{acq}} = \frac{C_{AB}}{\sigma_0}\sqrt{n_{voxel}}$$

- Resolution -> σ_0 -> n_{voxel} -> $\mathcal{V}_{AB}$(?)
- Partial volumed voxel contains >1 tissues/objects
- Partial volume effect (PVE) is inevitable in in-vivo MR imaging
  - Finite voxel size (0.25mm ~ 4mm)
  - Complex tissue boundaries
  - Micro vasculature/structures
  - …

CNR, partial volume and resolution

- Size of A: $\alpha\Delta x$, $(\alpha<1)$
- Size of B: Δx or $\beta\Delta x$, $(\beta<1)$
- In both scenarios: $C_{AB} = \alpha(S_A - S_B)$
- Noise σ: $\sigma(\alpha) = \sigma_0, \quad (\alpha L_x, N)$

$$\sigma(\alpha) = \sigma_0/\sqrt{\alpha}, \quad (L_x, \alpha N)$$

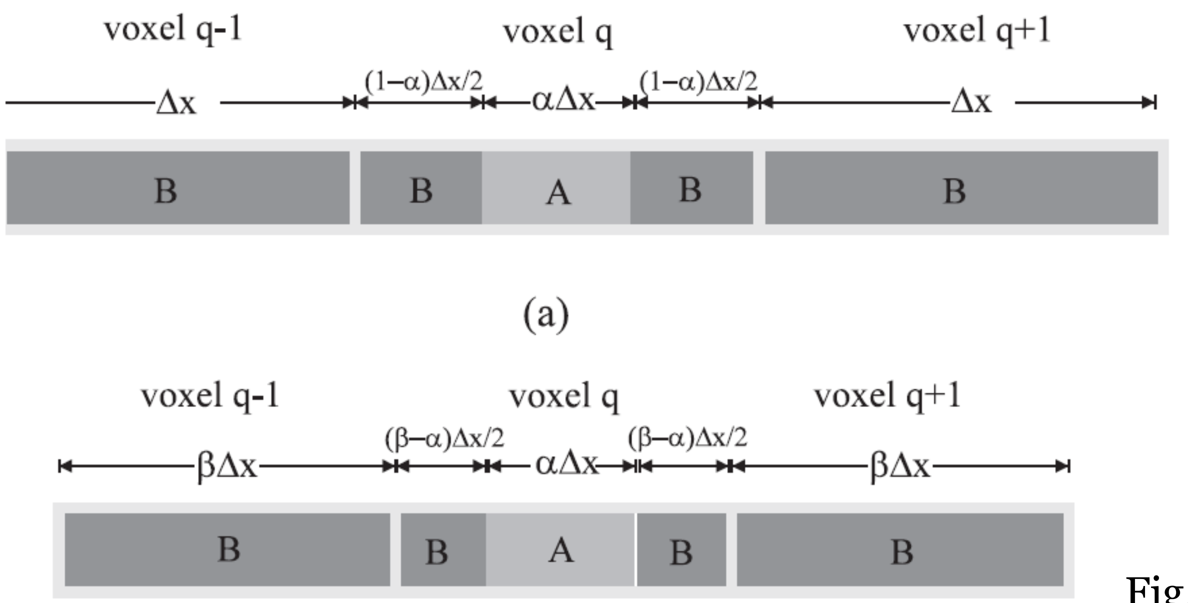
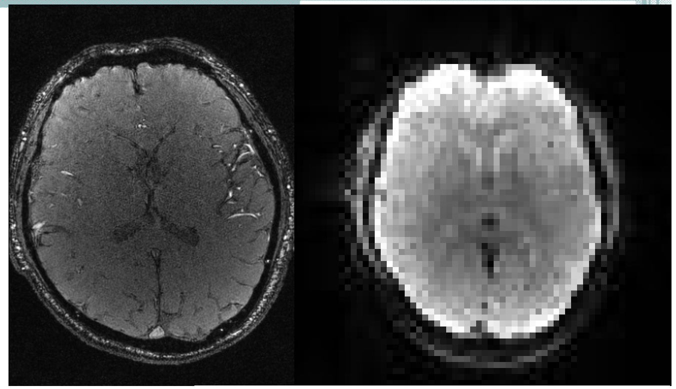
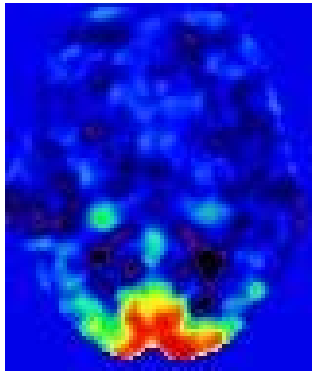


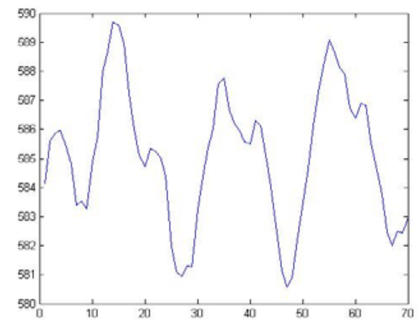
Fig. 15.13

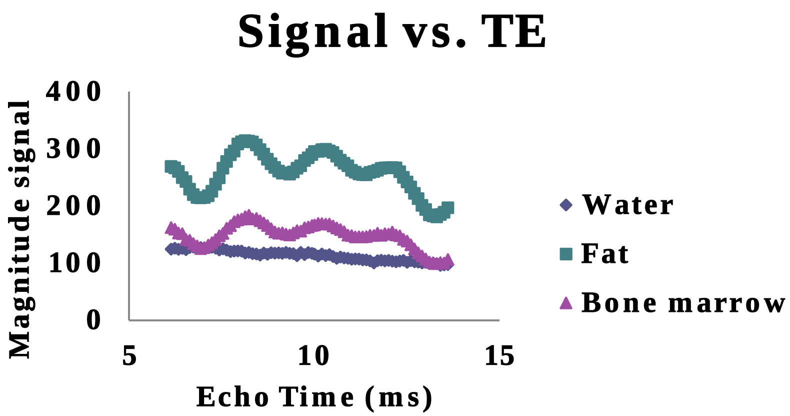
How PVE affects the image?

- Blurring
- Signal dephase (T2'/T2*, BOLD imaging, chap 25)
- Double/multiple T2/T2* exponential decay
- Signal cancellation (water/fat)

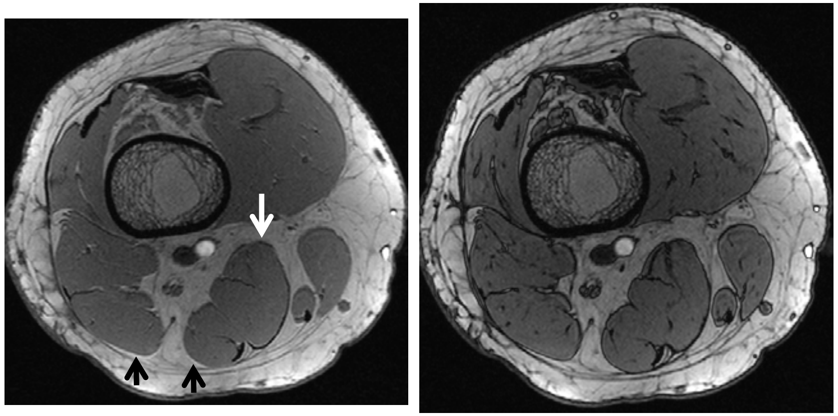








Note the similarity with Fig. 17.8
Courtesy of Manju



Signal and noise of phase images

- S_{phase} is affected by chemical shift, local susceptibility and/or motion, offering different contrast than magnitude images
- S_{phase} is a relative value since baseline $\phi_0$is unknown
- Only the contrast in phase has useful information
- $\sigma_{phase} = 1/SNR_{mag}$
- Phase may still provides sufficient contrast when magnitude fails

Phase imaging applications

- Chemical shift (fat/water) imaging (Chap. 17)
- B_0 field homogeneity mapping (Chap. 20)
- Phase contrast MR angiography (Chap. 24)
- Blood flow quantification (Chap. 24)
- Susceptibility Weighted Imaging (SWI, Chap. 25)
- …

Homework

- Prob 15.1, 15.3, 15.4, 15.8, 15.10, 15.12, 15.13

Next Session

Chapter 15.3-15.6

Most important contrast types

- Examples:

T1 values at 3T
GM : ~1200ms
WM: ~800ms
CSF: ~2-4s